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R. M. BEVENSEE

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# GROUNDWAVE PROPAGATION OVER VERY ROUGH TERRAINS\*

R. M. Bevensee

(Lawrence Livermore National Laboratory)

(University of California, Livermore, CA 94550 USA)

## ABSTRACT

A very powerful method of predicting single-frequency  $TM_p$  ground wave propagation over irregular and/or inhomogeneous terrain is by a Volterra integral equation, based on a Compensation Theorem for inhomogeneities. Such a treatment utilizes terrain elevation and surface impedance profiles and neglects backscatter and variations transverse to the propagation path. For very rough terrains, the computed relative field (attenuation) function may become physically unrealistic, indicating the need to include backscatter. For this situation a modified Volterra treatment with a first-order Fredholm integral equation correction is appealing.

## INTRODUCTION

The geometry of a distributed antenna around an origin  $O$  above an irregular and perhaps inhomogeneous terrain of profile  $z(x)$  is shown in Fig. 1. The  $TM_p$  field for either vertical current sources or horizontal sources aligned with the propagation direction  $x$  has  $E_p = E_x, E_z$ , and  $H_\phi = H_y$  components. The earth of complex relative dielectric constant  $\epsilon_g$  is assumed flat and homogeneous, with  $|\epsilon_g| \geq 9$  directly under the origin and extending a distance  $x \geq 2\lambda_0$  from any radiating element. These conditions ensure accuracy in computing the relative field factor  $f_A(x)$ , defined in

$$H_\phi(x) = 2H_0(x) f_A(x) \quad (1)$$

with

$$H_0(x) = jke^{-jkx}/4\pi x \quad (2)$$

the free-space field of a unit vertical current moment. For horizontal current sources, the factor  $(-\Delta_r)$  is appended to  $H_0(x)$ .  $\Delta_r$  is the reference normalized surface impedance

$$\Delta_r = -\sqrt{\epsilon_0/\mu_0} E_x/H_y = \sqrt{\epsilon_g-1/\epsilon_g} \quad (3)$$

directly under the antenna. For inhomogeneous terrain  $f_A$  is "compensated" for the fact that  $\Delta(x) \neq \Delta_r$ . Note  $|\Delta_r|^2 \ll 1$  for  $|\epsilon_g| \geq 9$ .

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The large-distance assumptions which validate the subsequent computations of  $f_A(x)$  over irregular terrain are (A)  $kx \gg 1$ ,  $k = 2\pi/\lambda_0$ , and  $h_a/x \ll |\Delta_r|$ .

#### OTT-VOLTERRA INTEGRAL EQUATION FOR IRREGULAR, HOMOGENEOUS TERRAIN

Ott and Berry<sup>1</sup> and Ott<sup>2</sup> wrote  $H_\phi(x) = \partial P / \partial x$  with  $P = \exp(-jkx)\psi(x)$ , and neglected  $\partial^2 \psi / \partial x^2$ -term (parabolic wave equation approximation). Their Volterra integral equation for  $f_A = f_1$  over a terrain profile of  $\eta(\xi)$  is (exp(j $\omega t$ ) time variation)

$$f_1(x) = f_{10}(0, x) - \sqrt{\frac{j}{\lambda}} \int_0^x f_1(\xi) e^{-jk\omega(\xi, x)} \left[ \frac{\partial \eta}{\partial \xi} W(\xi, x) - \frac{z(x) - \eta(\xi)}{x - \xi} \right] \cdot \sqrt{\frac{x}{\xi(x - \xi)}} d\xi \quad (4)$$

where  $f_{10}(0, x)$  is the direct source term

$$f_{10}(0, x) = \sum_i \int I_i(\bar{r}_i) dl_i e^{jkx_i} \exp\left[jk(2z - z_i) \frac{z_i}{2x}\right] W(x_i, x), \quad (5)$$

$$\omega(\xi, x) = \frac{1}{2} \left[ \frac{(z(x) - \eta(\xi))^2}{x - \xi} + \frac{\eta^2(\xi)}{\xi} - \frac{z^2(x)}{x} \right] \quad (6)$$

and  $W(\xi, x)$  is the *modified Sommerfeld attenuation function*,

$$W(\xi, x) = 1 - j\sqrt{\pi p} w(-\sqrt{u}) \quad (7)$$

$$w(-\sqrt{u}) = e^{-u} \operatorname{erfc}(j\sqrt{u}),$$

$$p = -j\frac{1}{2}k \Delta_r^2 (x - \xi) \quad (8)$$

$$u = -j\frac{1}{2}k \left( \Delta_r - \frac{z(x) - \eta(\xi)}{x - \xi} \right)^2 (x - \xi), \quad \xi < x \quad (9)$$

#### COMPENSATION-THEOREM VOLTERRA EQUATION FOR INHOMOGENEOUS TERRAIN

To "compensate" for  $\Delta(x) \neq \Delta_r$  and obtain  $f_A(x_0)$  we first compute  $f_1(x)$  out to  $x_0$  by solving (4) for successively larger  $x$ . Then we compute  $f_2(x \leq x_0)$  for a vertical test dipole at  $x_0$  by solving (4) in the *backward*

direction, with  $f_{10}(0, x_2 = x_0 - x) = W(0, x_2)$ . Then we solve for  $f_A(x_0)$  from the Volterra equation

$$f_A(x_0) = f_1(x_0) - \sqrt{\frac{j}{\lambda}} \int_0^{x_0} [\Delta(\xi) \sqrt{1 + (\partial\eta/\partial\xi)^2} - \Delta_r] f_A(\xi) f_2(x_0 - \xi) d\xi. \quad (10)$$

This equation requires knowledge of intervening  $f_A(\xi < x_0)$ , obtained by solving (10) at points  $x < x_0$  and perhaps extrapolating to  $x_0$ .

In this rather computationally expensive manner we obtain the relative field  $f_A(x)$  for inhomogeneous terrain, either flat or irregular.

#### A VOLTERRA-FREDHOLM TREATMENT TO INCLUDE BACKSCATTER (HOMOGENEOUS PATHS)

On some very rough paths the effective surface impedance within the brackets of (9) enters quadrant II of the complex plane. Then  $W(\xi, x)$  of (7) behaves as  $\exp[b^2 - a^2]$ , where  $-\sqrt{u} = a + jb \propto |x - \xi|$ , and backscatter should be included.

The kernel  $K(\xi, x)$  for  $\xi \leq x$  in (4) represents propagation forward. The analogous "backward" kernel  $K_+(\xi, x)$  for  $\xi > x$  is of the same form, but with the extra phase factor  $\exp[-jk_2(\xi - x)]$ ,  $(x - \xi)$  replaced by  $(\xi - x)$ , and  $(\partial\eta/\partial\xi) W(\xi, x)$  replaced by  $-(\partial\eta/\partial\xi) W_+(\xi, x)$ , where  $W_+$  is  $W$  of (7) with  $(x - \xi)$  replaced by  $(\xi, x)$ .

The following procedure is based on solving (4) up to  $x = x_v$ , with  $x_v$  determined from an error criterion, followed by corrections to  $f_1(\xi < x_v)$  and a first-order estimate of  $f(\xi > x_v)$ .

While stepping (4) forward through successively larger  $x$ , with  $f_1 = f_v$ , the Volterra solution, we also compute  $(\alpha = \sqrt{j/\lambda})$

$$-\alpha \int_x^{x_M} f_{10}(\xi) K_+ d\xi = E(x), \text{ error estimate.} \quad (11)$$

$x_M$  is the furthest contributing point on the terrain. When  $|E(x)| \approx 0.1 \cdot |f_v(x)|$  we set that  $x = x_v$ .

Then for any  $\xi_1 < x_v$  we write the improved solution,  $f_1$  as

$$f_1(\xi_1) = f_v(\xi_1) + f_a(\xi_1), \quad (12)$$

where  $f_a$  is obtained from the Volterra equation

$$0 \leq \xi_1 \leq x_v, \quad f_a(\xi_1) = -\alpha \left[ \int_0^{\xi_1} f_a(\xi) K d\xi + \int_{\xi_1}^{x_v} f_v(\xi) K_+ d\xi + \int_{x_v}^{x_M} f_{10}(\xi) K_+ d\xi \right]. \quad (13)$$

For any  $\xi_2 > x_v$  we write the solution as

$$f(\xi_2) \approx f_{10}(\xi_2) + f_b(\xi_2) \quad (14)$$

where  $f_b$  is obtained by the integral

$$x_M \geq \xi_2 > x_v, f_b(\xi_2) = -\alpha \left[ \int_0^{x_v} f_v(\xi) K d\xi + \int_{x_v}^{\xi_2} f_{10}(\xi) K d\xi + \int_{\xi_2}^{x_M} f_{10}(\xi) K d\xi \right]. \quad (15)$$

As  $\xi_1 \rightarrow x_v \leftarrow \xi_2$ ,  $f_1(\xi_1)$  of (12)  $\rightarrow f(\xi_2)$  of (14) (continuity).

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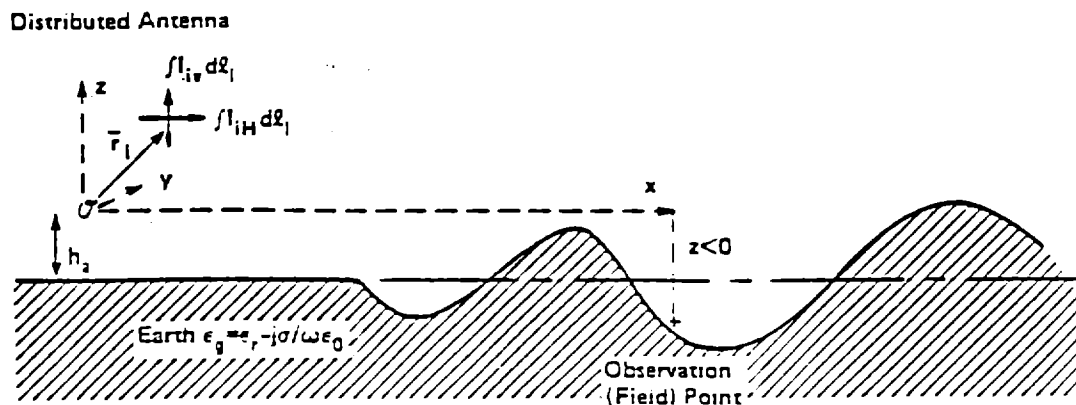


Fig. 1. Geometry of a distributed (test) antenna near origin 0 in a cartesian coordinate system. Surface waves for V- or H-polarized current sources are studied in the direction x.  $\int I_i(\vec{r}_i) d\ell_i$  is an elemental current moment centered at  $\vec{r}_i$ .